T 8228

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Third Semester

Civil Engineering

MA 1201 — MATHEMATICS — III

(Common to all branches Except Bio-Medical Engineering and B.E. (Part-Time) Second Semester Regulation 2005)

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 x 2 = 20 marks)

1. Form the partial differential equation of all spheres whose centres lie on the z-axis.

2. Find the complete integral of the partial differential equation $(1-x)p+(2-y)q=3-z$.

3. Find the value of a_0 in the cosine series expansion of $f(x)=K$ in the interval $(0,10)$.

4. Find the root mean square value of the function $f(x)=x$ in the interval $(0,l)$.

5. State one dimensional heat equation with the initial and boundary conditions.

6. Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement $f(x)$ and initial velocity $g(x)$.

7. Find the Fourier cosine transform of $f(x)$ defined as

$$f(x)=\begin{cases} x, & \text{for } 0<x<1 \\ 2-x, & \text{for } 1<x<2 \\ 0, & \text{for } x>2. \end{cases}$$

8. Rove that $F[f(x-a)] = e^{ias} F(s)$.

9. Find the z-transform of $(n+2)$.

10. State the find value theorem in z-transform.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular integral of $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Solve $(D^2 - 5DD' + 6D'^2)z = y \sin x$. (8)

Or

(b) (i) Solve $(3z - 4y)p + (4x - 2z)q = (2y - 3x)$. (8)

(ii) Solve $(D^2 + 2DD' + D'^2)z = x^2 y + e^{x-y}$. (8)

12. (a) Find the Fourier series expansion of period l for the function

$$f(x) = \begin{cases} x, & \text{in } \left(0, \dfrac{l}{2}\right) \\ l - x, & \text{in } \left(\dfrac{l}{2}, l\right). \end{cases}$$

Hence deduce the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$. (16)

Or

(b) (i) Find the half range cosine series of $f(x) = (\pi - x^2)$ in the interval $(0, \pi)$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots + \infty$. (8)

(ii) Find the Fourier series as the second harmonic to represent the function given in the following data : (8)

x :	0	1	2	3	4	5
y :	9	18	24	28	26	20

13. (a) A tightly stretched string of length $2l$ is fixed at both ends. The midpoint of the string is displaced by a distance "b" transversely and the string is released from rest in this position. Find the displacement of any point of the string at any subsequent time. (16)

Or

(b) An infinitely long uniform plate is bounded by two parallel edges and an end at right angle to them. The breadth of this edge $x=0$ is π, this end is maintained at temperature as $u=K(\pi y-y^2)$ at all points while the other edges are at zero temperature. Find the temperature $u(x,y)$ at any point of the plate in the steady state. (16)

14. (a) (i) Find the Fourier Sine transform of the function $f(x)=\frac{e^{-ax}}{x}$. (8)

(ii) Use transform method to evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)}$.

Or

(b) Find Fourier Cosine transform of $e^{-a^2x^2}$ and hence find Fourier sine transform of $xe^{-a^2x^2}$. (16)

15. (a) (i) Find $z^{-1}\left(\frac{z(z^2-z+2)}{(z+1)(z-1)^2}\right)$ by using method of partial fraction. (6)

(ii) Using z-transform solve difference equation $y(n+2)-4y(n+1)+4y(n)=0$ given that $y(0)=1$ and $y(1)=0$. (10)

Or

b (i) Find the z transform of $\frac{1}{(n+1)(n+2)}$. (8)

(ii) Using convolution theorem evaluate $z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$. (8)